Improvement in Calculation of Some Surface Integrals: Application to Junction Characterization in Cavity Filter Design

Philippe Guillot, Patrick Couffignal, Henri Baudrand, *Senior Member, IEEE,* and Bernard Theron

Abstract—**An integral method is presented for the accurate characterization of waveguide junctions which are commonly used in the design of cavity filters. The specific feature of this approach is a reduction of a surface integral to a simple contour integral and the consequent reduction in the computation time of the junction scattering parameters is more than 50%. The method is applied to calculate the electromagnetic coupling between circular cavities through rectangular irises as well as their coupling to the input and output waveguides. This method, in conjunction with an optimization procedure, is employed for the direct design of a dual mode cavity filter and the obtained results are in good agreement with experimental data.**

Introduction

MULTIMODE cavity filters are widely used in microwave telecommunication systems. In order to increase the order of these filters, several cavities should be used in cascade and the electromagnetic coupling between the adjacent cavities is achieved by means of rectangular or cross irises [1]–[5]. These irises can also be used to ensure the coupling of the filter to the input and output waveguides. In this way, the design of these filters will largely depend on the determination of the different coupling coefficients and consequently, an accurate characterization of the coupling irises constitutes the basis of the whole design. After having calculated the scattering parameters of each discontinuity, the total transmission through the structure can be determined by considering the cascaded discontinuities [8]–[10].

Several formulations have been used to study discontinuities. Finite differences, finite elements and TLM are general methods which require important computation facilities. The generalized S matrix method based on the development of electromagnetic fields at the junction is often used [6], [7]. Recently, a moment method formulation with entire basis and testing functions was used to analyze the transition between rectangular and circular waveguides coupled by a rectangular slot [11].

Manuscript received March 26, 1993; revised June 18, 1993.

P. Guillot, P. Couffignal, and H. Baudrand are with Laboratoire d'Electronique, ENSEEIHT, 2 rue Camichel, 31071 Toulouse Cedex, France.

B. Theron is with Alcatel Espace, 26 avenue Champollion, 31037 Toulouse Cedex, France.

IEEE Log Number 9212966.

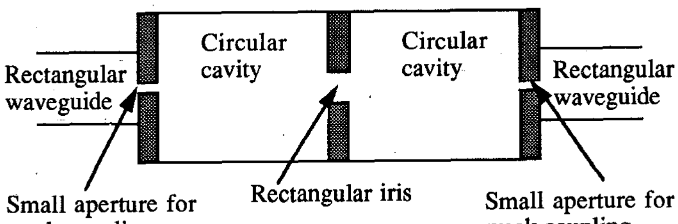


Fig. 1. Coupling between two circular cavities by rectangular iris. Measurement structure.

In the present work, the modeling of different discontinuities is based on a variational integral multimodal method. The junctions are then cascaded by taking into account the number of coupled modes [8]–[10]. This approach provides accuracy and small size matrices, thus rapidity. However, the field continuity condition at the junction involves scalar products calculation over the surface of the aperture, so that a surface integral must be calculated. In the case of a circular to rectangular transition [12], this integral is carried out numerically. Indeed, the expressions of the electromagnetic fields in the circular waveguide involve Bessel functions.

In this paper, a rigorous method is developed, allowing the reduction of a surface integral to a contour integral. This method will be applied to the study of the circular to rectangular discontinuity. The variation of the coupling between two circular cavities by a rectangular iris is studied for several iris lengths (Fig. 1). Our results are compared with experimental data. According to this design approach, a six-pole elliptic filter has been realized and the filter response has been found to be in good agreement with the initial requirements.

Theory

Suppose a surface S limited by a contour L (Fig. 2), where the quantity I is calculated by the following integral

$$I = \int_S (f_x g_x + f_y g_y)\, dS \tag{1}$$

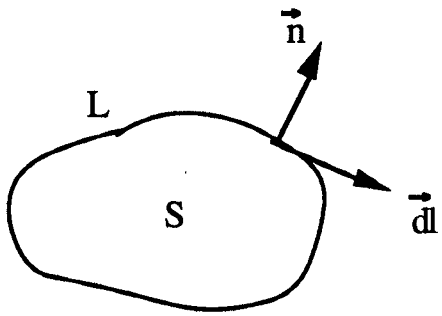


Fig. 2. Surface S limited by a contour L.

with,

$\vec{dl}$ = unit vector tangent to the aperture contour L.
$\vec{u}$ = unit vector normal to dl and in the plane of the aperture.

Here f and g are functions of two variables with two components, deriving from functions e, h, E, and H (scalar functions of two variables).

$$\begin{aligned} f_x &= \partial_x E - \partial_y H \qquad g_x = \partial_x e - \partial_y h \\ f_y &= \partial_y E + \partial_x H \qquad g_y = \partial_y e + \partial_x h. \end{aligned} \tag{2}$$

Therefore, the quantity I can be written as a sum of four integrals

$$I = I_{(1)} + I_{(2)} + I_{(3)} + I_{(4)} \tag{3}$$

where the integral expressions are given by

$$I_{(1)} = \int_s [\partial_x E\, \partial_x e + \partial_y E\, \partial_y e]\, dS \tag{4}$$

$$I_{(2)} = \int_s [\partial_y H\, \partial_y h + \partial_x H\, \partial_x h]\, dS. \tag{5}$$

$$I_{(3)} = \int_s [-\partial_y H\, \partial_x e + \partial_x H\, \partial_y e]\, dS \tag{6}$$

$$I_{(4)} = \int_s [-\partial_x E\, \partial_y h + \partial_y E\, \partial_x h]\, dS. \tag{7}$$

Moreover, we assume that the following relationships hold

$$\nabla^2 \begin{pmatrix} E \\ H \end{pmatrix} = \lambda \begin{pmatrix} E \\ H \end{pmatrix} \quad \nabla^2 \binom{e}{h} = \mu \binom{e}{h} \qquad \lambda \neq \mu. \tag{8}$$

Consider the following vector identity, where $\vec{A}$ is a vector and u a scalar function

$$\int_s \nabla \cdot (u\, \vec{A})\, dS = \int_s \nabla\, u \cdot \vec{A}\, dS + \int_s u\, \nabla \cdot \vec{A}\, dS. \tag{9}$$

On the other hand, when a volume V is flattened into a surface S, the Divergence Theorem can be expressed by the following limit identity

$$\int_s \nabla \cdot (u\, \vec{A})\, dS = \int_L u\vec{A} \cdot \vec{n}\, dl \tag{10}$$

this relationship holds for the surface S of Fig. 2, and (9) becomes

$$\int_s \nabla u \cdot \vec{A}\, dS = \int_L u\vec{A} \cdot \vec{n}\, dl - \int_s u\nabla \cdot \vec{A}\, dS. \tag{11}$$

Now by taking $u = e$ and $\vec{A} = \nabla E$, (11) becomes

$$\int_s \nabla E \cdot \nabla e\, dS = \int_L e\nabla E \cdot \vec{n}\, dl - \int_s e\nabla^2 E\, dS \tag{12}$$

the left hand side of the above equation is equal to $I_{(1)}$ and by using (8), it yields

$$I_{(1)} = \int_L e\, \nabla\, E \cdot \vec{n}\, dl - \lambda \int_s eE\, dS. \tag{13}$$

In the same way if we take $u = E$ and $\vec{A} = \nabla e$, we will obtain

$$I_{(1)} = \int_L E\nabla e \cdot \vec{n}\, dl - \mu \int_s Ee\, dS. \tag{14}$$

The combination of (13) and (14) gives

$$\int_s Ee\, dS = \frac{1}{(\lambda - \mu)} \int_L [e\nabla E - E\nabla e] \cdot \vec{n}\, dl. \tag{15}$$

Finally, the combination of (13) and (15) yields the following expression for $I_{(1)}$

$$I_{(1)} = \int_L \left[\left(\frac{\lambda}{\lambda - \mu} \right) E\nabla e - \left(\frac{\mu}{\lambda - \mu} \right) e\nabla E \right] \cdot \vec{n}\, dl. \tag{16}$$

In a similar way, and by taking successively $u = h$, $\vec{A} = \nabla H$ and $u = H$, $\vec{A} = \nabla\, h$ we will obtain the following expression for $I_{(2)}$

$$I_{(2)} = \int_L \left[\left(\frac{\lambda}{\lambda - \mu} \right) H\nabla h - \left(\frac{\mu}{\lambda - \mu} \right) h\nabla H \right] \cdot \vec{n}\, dl. \tag{17}$$

In order to calculate $I_{(3)}$, we will take $u = e$ and $\vec{A} = \binom{-\partial_y H}{\partial_x H}$ and (11) gives

$$\int_s \vec{A} \cdot \nabla e\, dS = \int_L e\vec{A} \cdot \vec{n}\, dl - \int_s e\nabla \cdot \vec{A}\, ds \tag{18}$$

by noting that the left hand side of the above relationship is equal to $I_{(3)}$ and that $\nabla \cdot \vec{A} = 0$, we obtain:

$$I_{(3)} = \int_L e(\nabla H \times \vec{n}) \cdot \vec{dl}. \tag{19}$$

Now by taking $u = E$ and $\vec{A} = \binom{-\partial_y h}{\partial_x h}$, the following expression is derived for $I_{(4)}$

$$I_{(4)} = \int_L E(\nabla h \times \vec{n}) \cdot \vec{dl} \tag{20}$$

and the quantity I will take the following form

$$I = \int_L \left\{ \frac{\lambda}{\lambda - \mu} (E\nabla e + H\nabla h) + \frac{\mu}{\mu - \lambda} \cdot (e\nabla E + h\nabla H) \right\} \cdot \vec{n}\, dl + \int_L (e\nabla H + E\nabla h) \times \vec{n} \cdot \vec{dl}. \tag{21}$$

It can be noted that each of the scalar functions e, h, E and H (denoted by φ) will satisfy the following relationships

$$\nabla\varphi \cdot \vec{n}\, dl = \frac{\partial \varphi}{\partial n} dl \quad \text{and} \quad \nabla\varphi \times \vec{n} \cdot \vec{dl} = \frac{\partial \varphi}{\partial l} dl. \tag{22}$$

Now, the above relationships can be applied to the study of the circular to rectangular waveguide discontinuity (Fig. 3). Then the integral is reduced according to the character of the modes, TE or TM, on each side of the discontinuity.

1) $TE_c - TE_r$ modes:

$$I = \int_L \frac{\mu}{\mu - \lambda} h \frac{\partial H}{\partial n} dl$$

2) $TM_c - TM_r$ modes:

$$I = \int_L \frac{\lambda}{\lambda - \mu} E \frac{\partial e}{\partial n} dl$$

3) $TM_c - TE_r$ modes:

$$I = \int_L E \frac{\partial h}{\partial l} dl$$

4) $TE_c - TM_r$ modes:

$$I = 0$$

Here, the functions E and H represent the electric and magnetic fields on the cavity side while e and h correspond to those of the rectangular side. They satisfy the Maxwell's equations as well as the continuity conditions in the junction plane.

The scalar products have been evaluated with the surface integral method as well as with the contour integral method. The error on the scalar products values is less than 10^{-3}%. These results validate our formulation. Fig. 4 shows the comparison between typical computation times required to obtain the same results by the two approaches (surface integral and contour integral). The contour integral method is seen to be two to three times faster. The application leads to a considerable time saving for the scalar product calculation and consequently for the scattering matrix calculation of the circular to rectangular junction.

The method used for the determination of the junction scattering parameters is a multimodal variational approach [8]–[9] which has the advantage of being accurate, fast, and where the use of small size matrices reduces the computer memory occupation.

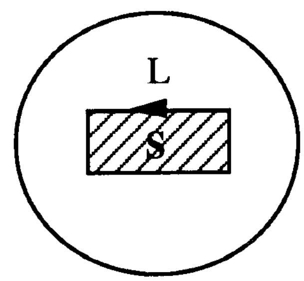


Fig. 3. Circular to rectangular waveguide junction.

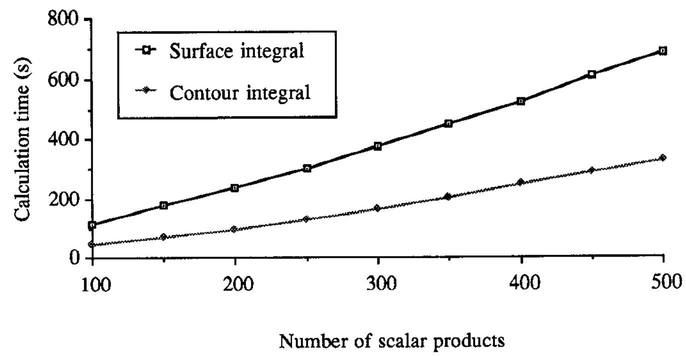


Fig. 4. Comparison of the calculation time according to the integration method for circular to rectangular junction.

Numerical and Experimental Results

This method is used to calculate the coupling between two circular cavities by a rectangular iris. The structure used for measuring the coupling between cavities is shown in Fig. 1. The cavities are coupled to the input (or output) waveguides through small apertures in order to ensure weak couplings. The transmission of the structure is characterized by means of the bandwidth of the frequency response (Fig. 5).

In practice, thin irises are used where their lengths are determined in function of the desired bandwidth. The input waveguide is excited by the fundamental mode TE_{10}, which is the only propagating mode. So, only $TEmn$ and $TMmn$ modes, with m odd and n even for rectangular waveguide and m odd for Bessel functions Jm of circular waveguides are considered.

In order to ensure a good numerical convergence of the bandwidth with the mode number in both waveguides (circular waveguide and rectangular iris), the bandwidth has been determined by using different numbers of modes. The diameter and length of the cylindrical cavities are respectively 26.92 mm and 45.395 mm. Fig. 6(a) shows the convergence of the numerical results when the number of modes in the rectangular iris is increased, while Fig. 6(b) illustrates the convergence in terms of number of modes in the circular waveguide. Both variations are shown for several iris lengths. According to these variations, 350 circular modes in the cavities and 10 modes in the iris will be sufficient for obtaining convergent values. Another

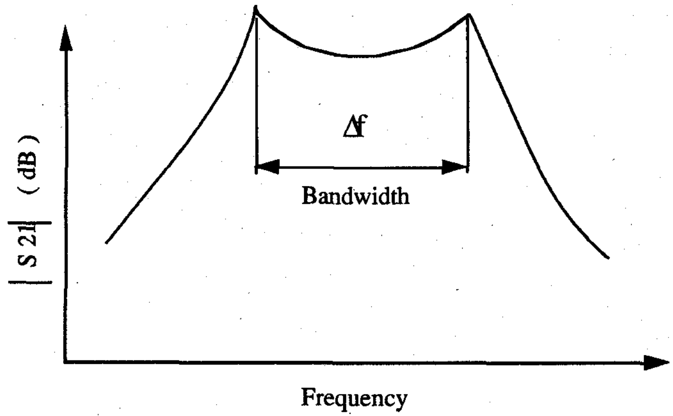


Fig. 5. Transmission response of two coupled cavities f_0: central frequency Δf: bandwidth.

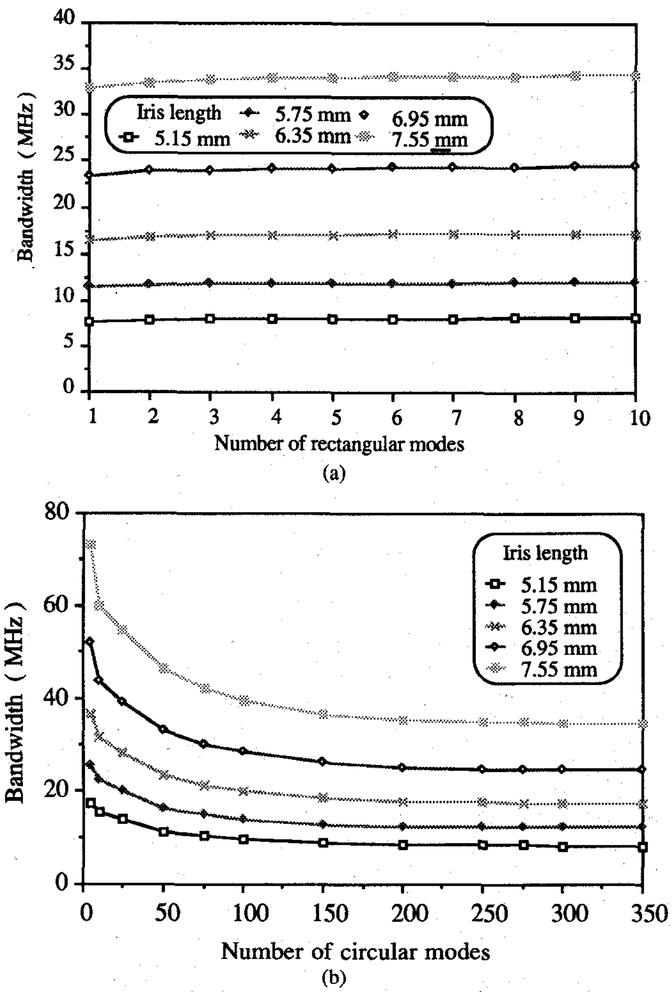


Fig. 6. (a) Convergence test for the bandwidth as a function of number of modes in the iris. (b) Convergence test for the bandwidth as a function of number of modes in the circular cavities.

convergence test has been made where three coupled modes have been necessary [8]–[10].

The bandwidth has been measured for several iris lengths. The diameter and length of the cylindrical cavities are, respectively, 26.46 mm and 42.4 mm. The iris length is varied as a parameter, while its width and thickness are fixed to 1 mm.

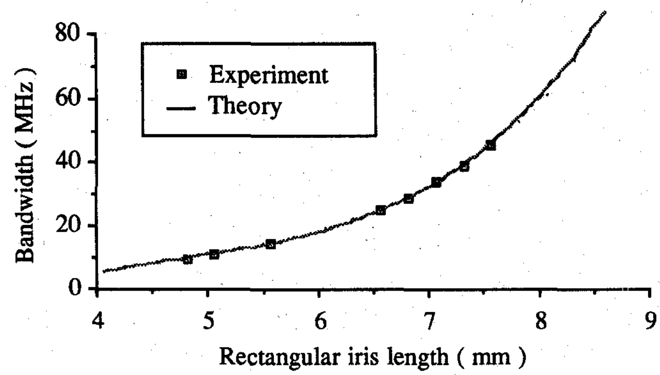


Fig. 7. Comparison between calculated and measured bandwidths obtained by two circular cavities coupled by a rectangular iris.

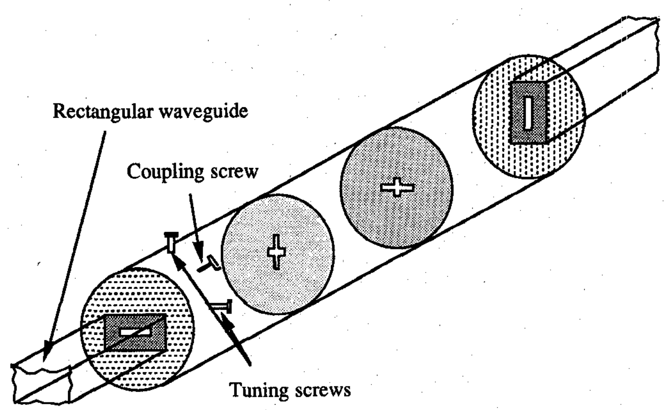


Fig. 8. Dual mode six-pole filter.

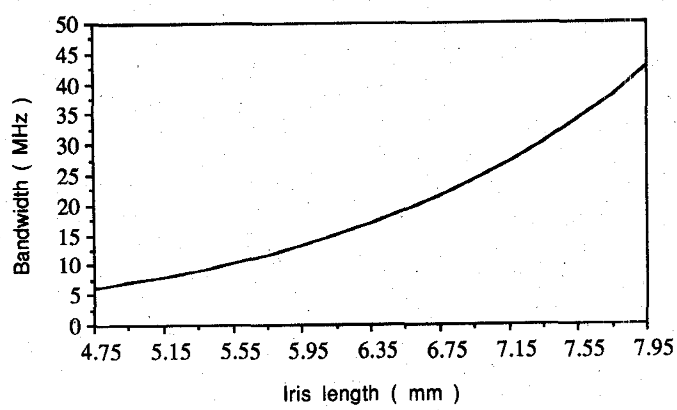


Fig. 9. Bandwidth as a function of the iris length (cross iris).

Then the theoretical variation of the bandwidth with the length of the rectangular iris has been calculated and presented in Fig. 7. The experimental data are in very good agreement with the theoretical results.

According to this design technique [13], a six-pole elliptic filter has been realized (Fig. 8). The required specifications are the following:

- central frequency: 11.85 GHz
- bandwidth: 40 MHz
- VSWR in the passband: <1.15

The bandwidth variation with the iris length is represented in Fig. 9. The dual mode cavity filter has been

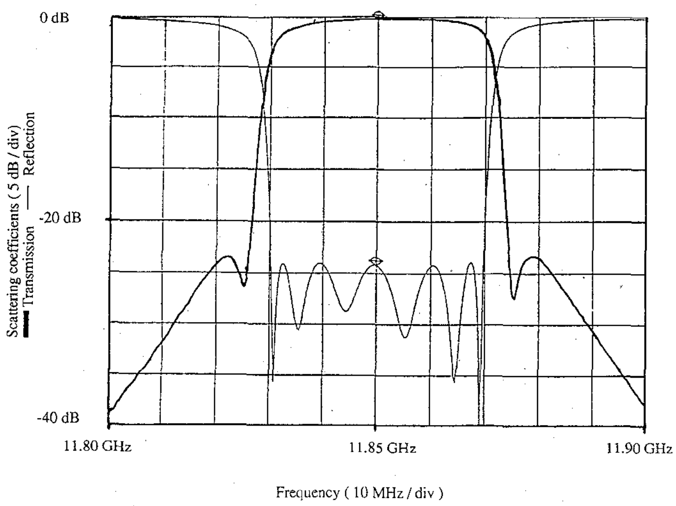


Fig. 10. Measured response of the six-pole elliptic filter. Required values: fo = 11.85 GHz, bandwidth 40 MHz, VSWR < 1.15.

realized using these values. The filter response has been measured (Fig. 10) and a very good agreement is observed with the predicted values. There has been no need for any further mechanical adjustments.

Conclusion

A rigorous method has been described to reduce a surface integral to a contour integral. The method validity has been demonstrated by studying a circular to rectangular junction. Moreover, our formulation exhibits a better performance and its gain in computation time is evident. This formulation associated with a multimodal variational method allows a full wave analysis of discontinuities and can be implemented on personal computers. This procedure has been applied to the design of a filter structure. According to the optimized computed data, filters have been realized by ALCATEL ESPACE. The measured values agree well with the predicted requirements.

References

[1] A. E. Williams, "A four cavity elliptic waveguide filter," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 1109–1114, Dec. 1970.

[2] A. E. Atia and A. E. Williams, "Narrow bandpass waveguide filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 258–265, April 1972.

[3] W. C. Tang and S. K. Chaudhuri, "A true elliptic function filter using triple mode degenerate cavities," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 1449–1454, Nov. 1984.

[4] R. R. Bonetti and A. E. Williams, "A TE triple-mode filter," *in 1988 IEEE MTT-S Int. Microwave Symp. Dig.*; pp. 511–514.

[5] U. Rosenberg and D. Wolk, "New possibilities of cavity filter design by a novel TE-TM mode iris coupling," *in 1989 IEEE MTT-S Int. Microwave Symp. Dig.*; pp. 1155–1158.

[6] H. Patzelt and F. Arndt, "Double plane steps in rectangular waveguides and their application for transformers, irises and filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 771–776, May 1982.

[7] F. Arndt, J. Bornemann, D. Heckmann, C. Piontek, H. Semmerow, and H. Schueler, "Modal S-matrix method for the optimum design of inductively direct coupled cavity filters," *IEE Proc.*, vol. MTT-133, Pt. H, pp. 341–350, October 1986.

[8] J. W. Tao and H. Baudrand, "Multimodal variation analysis of uniaxial waveguide discontinuities," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-39, pp. 506–516, March 1991.

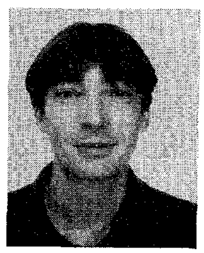
[9] J. W. Tao, H. Baudrand, B. Theron, and J. C. Nanan, "Analysis and design of ridged waveguide passband filter by variational approach," Annales des Telecommunications, 45, number 5-6, pp. 344–350, 1990.

[10] M. S. Navarro, T. T. Rozzi, and Y. T. Lo, "Propagation in a rectangular waveguide periodically loaded with irises," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 857–865, August 1980.

[11] B. N. Das and P. V. D. Somasekhar Rao, "Analysis of a transition between rectangular and circular waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-39, pp. 357–359, February 1991.

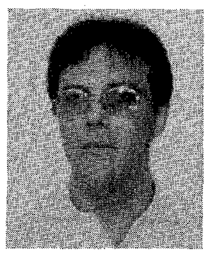
[12] B. N. Das, P. V. D. Somasekhar Rao, and A. Chakraborty, "Narrow wall axial slot coupled T-junction between rectangular and circular waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 1590–1596, October 1989.

[13] P. Couffignal, H. Baudrand, and B. Theron, "A new rigorous method for the determination of iris dimensions in dual mode cavity filters," to be published on *IEEE Trans. Microwave Theory and Tech.*



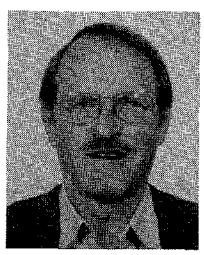
Philippe Guillot was born in Albi, France, in 1965. He received the DEA degree in electronics from ENSEEIHT, Toulouse, France, in 1990. He is currently working toward the Ph.D. degree.

Since 1990, he has been doing research work on waveguide discontinuity problems and microwave filter design in the Electronic Laboratory of ENSEEIHT.



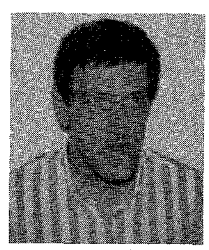
Patrick Couffignal was born in France in 1959. He received the Ph.D. degree from the Institut National Polytechnique of Toulouse, France, in 1992.

His research interest is in discontinuity modeling and the filter devices. Currently, he is doing postdoctoral research in the Electronics Laboratory of ENSEEIHT in Toulouse, France.



Henri Baudrand (M'86–SM'90) was born in France in 1939. He received the Diplome d'Ingénieur degree in electronics and the Docteur-ès-Science degree in microwaves, both from the Institut National Polytechnique of Toulouse, France, in 1962 and 1966, respectively.

Since 1966, he has been working on the modeling of active and passive microwave circuits by integral methods in the Electronics Laboratory of ENSEEIHT in Toulouse. Currently, he is a Professor of Microwaves and is in charge of the Microwaves Research Group.



Bernard Theron was born in France in 1950. He received the Diplome d'Ingénieur degree in electrical engineering in 1975.

From 1977 to 1978, he worked at CNET (French Telecommunications Laboratory) on PCM/PSK systems. In 1978, he joined the Thomson-CSF Microwave Links Division to work on antenna systems. From 1980 to 1984 he was an engineer at Thomson-CSF and then at the Alcatel Espace Microwave Laboratory. While there, he worked on the design and development of microwave filters for telecommunications satellites TELECOM1, TDE, TELE-X, INTELSAT VI, EUTELSAT-II and miscellaneous R&D programs. From 1985 to 1988, he was Manager of the Passive Microwave Laboratory at Alcatel Espace and was in charge of passive R&D studies. Currently, he is the manager of the Receivers and Filters Laboratory at Alcatel Espace, Toulouse, France.